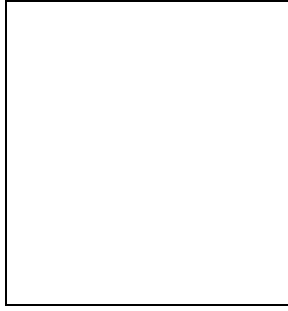# The Doppler peaks from a generic defect

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Abstract

We investigate which of the exotic Doppler peak features found for textures and cosmic strings are generic novelties pertaining to defects. We find that the "out of phase" texture signature is an accident. Generic defects, when they generate a secondary peak structure similar to inflation, apply to it an additive shift. It is not necessary for this shift to be "out of phase". We also show which factors are responsible for the absence of secondary oscillations found for cosmic strings. Within this general analysis we finally consider the conditions under which topological defects and inflation can be confused. It is argued that only $\Omega = 1$ inflation and a defect with a horizon size coherence length have a chance to be confused. Any other inflationary or defect model always differ distinctly.

1   Introduction

Recent work has addressed the hitherto virgin ground of Doppler peaks induced by motivated topological defect scenarios. Textures have been studied by [1] and [2] with good qualitative agreement, and cosmic strings were studied in [3, 4, 5]. This work prompts reflection on a more fundamental level. Inflationary perturbations have always been open to the most general class of possibilities. For instance one considered tilt, tensor modes, etc, long before motivated inflationary mechanisms were found to justify them. Defect theories on the contrary have always been tied down to concrete models produced by specific patterns of spontaneous symmetry breaking. Here we shall depart from this (highly positive) trend, and allow ourselves the same speculative freedom inflationary theories enjoy. We shall therefore ask what is a generic defect, and what are the generic Doppler peak features induced by a generic defect.

One metaphysical and one practical motivation assist this project. Firstly one would like to know how much of what has already been found for textures and strings are generic defect

novelties. For instance it was found in [1, 2] that texture Doppler peaks appear out of phase with respect to the standard inflationary peaks. Is this a robust defect feature? Or is it an accident pertaining to textures? In other words, could a generic defect apply an arbitrary shift to the Doppler peaks? This is an important question, as topological defects and standard isocurvature perturbations (like the ones discussed in [6]) are not at all the same thing, a fact often overlooked. Therefore there is no reason why an out of phase signature would have to be associated with defects. A second example is provided by cosmic strings. Despite many uncertainties it has been found that cosmic strings do not have secondary Doppler peaks. This is a rather exotic feature, completely alien to inflationary theories. Is this feature a robust prediction for a large class of defect models? And if so what are the controlling factors responsible for the opposite behaviour of strings and textures in this respect?

A second, perhaps more practical motivation for this type of work lies in the conflict between inflation and defects. Are the inflationary Doppler peaks proof of inflation, or could one in principle cook up a defect which reproduced the inflationary Doppler peaks? Until this question has been answered it is daylight robbery to claim that a measurement of, say, the CDM prediction for the C_ℓ spectrum, would prove inflation.

A detailed and more serious discussion of this problem may be found in [5]. Here we merely highlight the most entertaining aspects of this work.

2 The ontology of defects and inflation

We now focus on the basic assumptions of inflationary and defect theories and isolate the most shocking contrasting properties. We define the concepts of active and passive perturbations, and of coherent and incoherent perturbations. In terms of these concepts inflationary perturbations are passive coherent perturbations. Defect perturbations are active perturbations more or less incoherent depending on the defect.

2.1 Active and passive perturbations, and their different perceptions of causality and scaling

The way in which inflationary and defect perturbations come about is radically different. Inflationary fluctuations were produced at a remote epoch, and were driven far outside the Hubble radius by inflation. The evolution of these fluctuations is linear (until gravitational collapse becomes non-linear at late times), and we call these fluctuations "passive". Also, because all scales observed today have been in causal contact since the onset of inflation, causality does not strongly constrain the fluctuations which result. In contrast, defect fluctuations are continuously seeded by defect evolution, which is a non-linear process. We therefore say these are "active" perturbations. Also, the constraints imposed by causality on defect formation and evolution are much greater than those placed on inflationary perturbations.

2.1.1 Active and passive scaling The notion of scale invariance has different implications in these two types of theory. For instance, a scale invariant gauge-invariant potential Φ with dimensions $L^{3/2}$ has a power spectrum

$$P(\Phi) = \langle |\Phi_{\mathbf{k}}|^2 \rangle \propto k^{-3}$$

in passive theories (the Harrison-Zeldovich spectrum). This results from the fact that the only variable available is k, and so the only spectrum one can write down which has the right

dimensions and does not have a scale is the Harrison-Zeldovich spectrum. The situation is different for active theories, since time is now a variable. The most general counterpart to the Harrison-Zeldovich spectrum is

$$P(\Phi) = \eta^3 F_\Phi(k\eta) \tag{1}$$

where F_Φ is, to begin with, an arbitrary function of $x = k\eta$. All other variables may be written as a product of a power of η, ensuring the right dimensions, and an arbitrary function of x. Inspecting all equations it can be checked that it is possible to do this consistently for all variables. All equations respect scaling in the active sense.

2.1.2 Causality constraints on active perturbations

Moreover, active perturbations are constrained by causality, in the form of integral constraints [7, 8]. These consist of energy and momentum conservation laws for fluctuations in an expanding Universe. The integral constraints can be used to find the low k behaviour of the perturbations' power spectrum, assuming their causal generation and evolution [9]. Typically it is found that the causal creation and evolution of defects requires that their energy ρ^s and scalar velocity v^s be white noise at low k, but that the total energy fluctuations' power spectrum is required to go like k^4. To reconcile these two facts one is forced to consider the compensation. This is an underdensity in the matter-radiation energy density with a white noise low k tail, correlated with the defect network so as to cancel the defects' white-noise tail. When one combines the defects energy with the compensation density, one finds that the gravitational potentials they generate also have to be white noise at large scales [5]. Typically the scaling function $F_\Phi(k\eta)$ will start as a constant and decay as a power law for $x = k\eta > x_c$. The value x_c is a sort of coherence wavenumber of the defect. The larger it is the smaller the defect is. For instance $x_c \approx 12$ for cosmic strings (thin, tiny objects), whereas $x_c \approx 5.5$ for textures (round, fat, big things). Sophisticated work on causality [8] has shed light on how small x_c may be before violating causality. The limiting lower bound $x_c \approx 2.7$ has been suggested.

Although we will not here have a chance to dwell on technicalities, it should be stated that the rather general discussion presented above is enough to determine the general form of the potentials for active perturbations. This has been here encoded in the single parameter x_c. We shall see that x_c will determine the Doppler peak position for active perturbations. Doppler peaks are driven by the gravitational potential, so it should not be surprising that the defect length scale propagates into its potential, and from that into the Doppler peaks' position.

2.2 Coherent and incoherent perturbations

Active perturbations may also differ from inflation in the way "chance" comes into the theory. Randomness occurs in inflation only when the initial conditions are set up. Time evolution is linear and deterministic, and may be found by evolving all variables from an initial value equal to the square root of their initial variances. By squaring the result one obtains the variables' variances at any time. Formally this results from unequal time correlators of the form

$$\langle \Phi(\mathbf{k}, \eta)\Phi(\mathbf{k}', \eta') \rangle = \delta(\mathbf{k} - \mathbf{k}')\sigma(\Phi(k, \eta))\sigma(\Phi(k, \eta')), \tag{2}$$

where σ denotes the square root of the power spectrum P. In defect models however, randomness may intervene in the time evolution as well as the initial conditions. Although deterministic in principle, the defect network evolves as a result of a complicated non-linear process. If there is strong non-linearity, a given mode will be "driven" by interactions with the other modes in a way which will force all different-time correlators to zero on a time scale characterized by the "coherence time" $\theta_c(k, \eta)$. Physically this means that one has to perform a new "random"
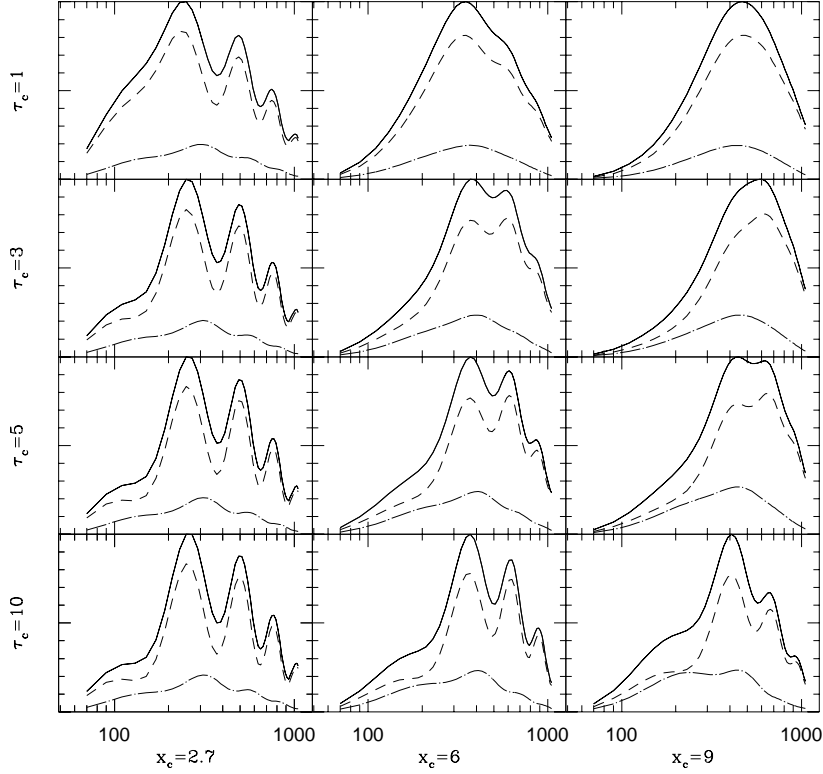
Figure 1: C_l spectra for a grid of models with various values of x_c (related to the defect coherence length) and $\theta_c \approx 2.35\tau_c$ (the defect coherence time). We have included the monopole term (dash) and dipole term (point-dash), Silk damping, and free-streaming. The monopole term is always dominant.

draw after each coherence time in order to construct a defect history [3]. The counterpart to (2) for incoherent perturbations is

$$\langle \Phi(\mathbf{k},\eta)\Phi(\mathbf{k}',\eta') \rangle = \delta(\mathbf{k}-\mathbf{k}')P(\Phi(k,\eta),\eta'-\eta) \ . \tag{3}$$

For $|\eta'-\eta| \equiv |\Delta\eta| > \theta_c(k,\eta)$ we have $P(\Phi(k,\eta),\Delta\eta) = 0$. For $\Delta\eta = 0$, we recover the power spectrum $P(\Phi(k,\eta),0) = P(\Phi(k,\eta))$.

We shall label as coherent and incoherent (2) and (3) respectively. This feature does not affect the Doppler peaks' position but it does affect the structure of secondary oscillations. An incoherent potential will drive the CMB oscillator incoherently, and therefore it may happen that the secondary oscillations get washed out as a result of incoherence.

3 Generic defect Doppler peaks

In Fig. 1 we show a grid of C_ℓ spectra functions of the two parameters introduced above: x_c (related to the defect coherence length) and θ_c (the defect coherence time). For the exact form of the stress energy of these defects we refer the reader to [5].

3.1 The peaks position

In general there may or may not be a system of secondary Doppler peaks. However if they exist, then their position is determined purely by x_c. For $x_c \approx 2.7$ (not impossible, but probably unrealistic because it is very close to the smallest turnover point allowed by causality [8, 10]) the peaks are at the adiabatic positions. As x_c increases from the adiabatic position the peaks are shifted to smaller scales. For $x_c \approx 5.4$ they are out of phase with the adiabatic peaks (as in [1, 2]). For $x_c > 8.5$ the peaks start only in the adiabatic secondary peaks region. For standard values of Ω_b and h these three cases would place the main "Doppler peak" at $l \approx 230$, 350, and 500, respectively. Therefore the placing of the peaks is *not* a generic feature of active fluctuations. Active perturbations simply add an extra parameter on which the Doppler peaks position is strongly dependent. In general we should expect that for the same Ω, Ω_b, and h, active perturbations will apply to the predicted CDM adiabatic peak position a shift of the form

$$l \to l + \frac{\eta_0}{\eta^*}\left(x_c - \frac{\pi\sqrt{3}}{2}\right) \tag{4}$$

where η_0 and η^* are the conformal times nowadays and at recombination. The secondary peaks' separation is not changed, in a first approximation. This is to be contrasted with non-flat inflationary models where $C_l(\Omega = 1)$ is taken into $C_{l\Omega^{-1/2}}$. The defect shift is additive whereas the low-Ω shift is multiplicative, a striking difference that should always allow us to distinguish between low Ω CDM and $\Omega = 1$ high-x_c defects.

3.2 Intensity of secondary oscillations

The strength of the secondary oscillations depends on both x_c and θ_c. For $x_c \approx 2.7$ there are secondary oscillations regardless of the exact θ_c value. This is a confusing defect, as not only does it place the Doppler peaks on the adiabatic position, but also the peak structure is quite insensitive to the defect incoherence. For larger x_c the secondary Doppler peaks survive only if the defect coherence time is much larger than x_c. This condition seems unphysical for large x_c so we expect realistic defects with large x_c not to have secondary oscillations.

This can be understood heuristically. Incoherence tends to erase secondary oscillations. However each mode is active only for a period of scaling time of the order of x_c. If $\theta_c > x_c$ then each mode is coherent for longer than it is active, and so the defect is effectively coherent and the secondary oscillations are preserved. If on the contrary $\theta_c < x_c$ then the defect has time to display its incoherence. Large defects (small x_c) are not active for long enough to display whatever reasonable incoherent properties they may have. Very small defects (large x_c) are active for long enough for their incoherence to be manifest, whatever reasonable coherence time they may have.

To help the reader to connect this general discussion with concrete defect theories, here is a rough guide to the topography of Fig. 1. Current understanding places the cosmic string models on the top right corner of Figure 1 (large x_c, τ_c smaller than 3). They should have a single peak well after the main adiabatic peak. Textures fall somewhere in the middle of the figure (x_c around 6, coherence time not yet measured). Their main peak should be out of phase with the adiabatic peaks. This is an accident related to the x_c value for textures, and not a robust defect feature. Texture secondary oscillations should exist but be softer than predicted by the coherent approximation (used in [1, 2]). How much softer depends on the exact value of the texture's θ_c. If their coherence time is of the same order as strings ($\tau_c \approx 3$) their secondary oscillation will be very soft.

4 Confusing defects and inflation

Given this state of affairs what are the chances of confusing inflation and defect theories? The answer to this question depends on Ω, on the inflation side, and on x_c, on the defect side. We have shown how only $\Omega = 1$ inflation and $x_c \approx 2.7$ (the causal lower bound) have a chance to be confused. Any defect with a larger x_c is bound to cause great disarray in what has come to be expected from Doppler peaks by inflationary trends. Two novelties stand out. First, if defects preserve a structure of secondary peaks, then this tends to be obtained from the inflationary one by an additive shift in l, rather than a multiplicative shift (as it happens for low Ω inflation). Second, defects may erase the secondary peaks.

It remains as an open question the case $\Omega = 1$ inflation vs $x_c = 2.7$ defects. Our work, and also [10] suggests that indeed in this case one may confuse defects and inflation. The work in [6] seems to draw the opposite conclusion. One would hope that this issue is clarified in the not too distant future.

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